1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1,Class XI,)
Module Name/Title	Unit 6,Module 2,Acceleration due to gravity Chapter 8,Gravitation
Module Id	Keph_10802_eContent
Pre-requisites	Gravitational force, Kepler's laws, Universal law of gravitation, universal gravitational constant
Objectives	After going through this lesson, the learners will be able to • **Understand** 'Acceleration due to gravity (g) on the surface of earth' and on other planets and get an expression for 'g' • **Derive** expression for Variation of acceleration due to gravity with height from the surface of the earth. • Understand the derivation of expression showing Variation of acceleration due to gravity g with depth • Appreciate the Role of other factors affecting the value of g
Keywords	Acceleration due to gravity, variation of g with depth and altitude, shell theorem

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	Central Institute of Educational Technology, NCERT, New Delhi
Programme Coordinator	Dr. Mohd. Mamur Ali	Central Institute of Educational Technology, NCERT, New Delhi
Course Coordinator / PI	Anuradha Mathur	Central Institute of Educational Technology, NCERT, New Delhi
Subject Matter Expert (SME)	SmitaFangaria	PGT Physics Developer Anveshika Amity International School, Noida
Review Team	Associate Prof. N.K. Sehgal (Retd.) Prof. V. B. Bhatia (Retd.) Prof. B. K. Sharma (Retd.)	Delhi University Delhi University DESM, NCERT, New Delhi

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1. UNIT SYLLABUS

Unit VI: Gravitation

Chapter 8: Gravitation

Kepler's laws of planetary motion; universal law of gravitation.

Acceleration due to gravity and its variation with altitude and depth.

Gravitational potential energy and gravitational potential; escape velocity; orbital velocity of a satellite; Geo-stationary satellites.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 5 Modules

The above unit is divided into five modules for better understanding.

Module 1	• Gravitation • Laws of gravitation • Early studies • Kepler's laws
Module 2	• Acceleration due to gravity • Variation of g with altitude • Variation of g due to depth • Other factors that change g

Module 3	• Gravitational field • Gravitational energy • Gravitational potential energy • Need to describe these values
Module 4	• Satellites • India's satellite programme and target applications • Geo stationary satellites and Polar satellites • Escape velocity • India's space program
Module 5	• Numerical problems based on Gravitation

MODULE 2

3. WORDS YOU MUST KNOW

- **Gravitational force**: Force of attraction between two objects of some mass.

- **Kepler's laws:**

Law of orbits:

All planets move in elliptical orbits with the Sun situated at oneof the foci of the ellipse.

Law of Areas:

Thelinethatjoinsanyplanet tothesunsweepsequalareasinequal intervals of time.

Law of periods:

The Square of the time period of revolution of a planet isproportional to the cube of the semi-major axis of the ellipsetraced out by the planet.

- **Newton's Universal Law of gravitation**: It states that the gravitational force between two point masses is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

- **Principle of superposition**: If we have a collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses
- **Gravitational force**: Force of attraction between two objects of some mass.
- **Celestial bodies**: Stars, Planets, comets, asteroids etc.
- **Ellipse**: A regular oval shaped curve which is the locus of a point moving in a plane so that the sum of its distances from two other points (the foci) is constant,
- **Eccentricity of an ellipse**: It is the measure of deviation of the ellipse from circularity.
- **Areal velocity**: It is the rate at which area is swept out by a particle as it moves along a curved path. In Kepler's law of areas, the particle is the planet and curve is the orbit in which it moves around the sun.
- **Universal gravitational constant**: Denoted by the letter G, it is an empirical physical constant involved in the calculation of gravitational effects in Newton's law of universal gravitation. It is a universal constant with the value 6.673×10^{-11}N m^2 kg^{-2}.

4. INTRODUCTION

An object moving under the influence of a force (F) experiences acceleration (a)in accordance with Newton's second law of motion given by the relation below.

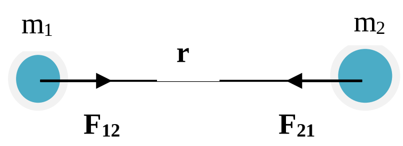
$$F = ma, \qquad \therefore a = \frac{F}{m}$$

The acceleration of the object is inversely proportional to its mass (m). Therefore, for a given force F, greater the mass of the object, smaller will be its acceleration. For example, if a table tennis ball and an iron ball of the same size were moving under the influence of the same force, the acceleration of the iron ball would be much smaller than the table tennis ball.

5. ACCELERATION DUE TO GRAVITY (G) ON SURFACE OF EARTH

Gravitational force acting between two particles of masses m_1 and $m_2$is mutual. By Newton's third law of motion

$$|F_{12}| = |F_{21}|$$



$\therefore m_1a_1 = m_2a_2$

Where a_1 and a_2 are accelerations of particle of mass m_1 and that of mass m_2 respectively.

Hence, the magnitude of the gravitational force on both the particles is the same, but the acceleration of particle with larger mass is smaller than the acceleration of the lighter particle.

For example: If $m_1 = 100m_2$, then $a_1 = a_2/100$

Suppose, mass $m_1$is the mass of the Earth. The Earth behaves as if the whole of its mass is concentrated at its center and hence the whole of the earth could be replaced by a point mass placed at its centre whose mass is equal to the mass of the earth(M).Here the assumption is that the earth is a perfect sphere of uniform density.

So by replacing mass m_1 by M(mass of earth)i.e $m_1 = M$ andthe distance r by R (radius of the earth) r = Rthe mass m_2 (replaced by m) is pulled towards the center of the earth by a force

$$F = \frac{GMm}{R^2}$$

The acceleration of the mass m towards the center of the earth is

$$\mathbf{a} = \frac{F}{m} = \frac{GM}{R^2} = \mathbf{g}$$

This acceleration is called the acceleration due to gravity g.

Value of g on the surface of the earth can be found out by substituting the value of G, M and R in the above equation

$$\mathbf{g = (6.67 \times 10^{-11} \times 5.98 \times 10^{24}) /(6.37 \times 10^{6})^2 = 9.8\ m/s^2}$$

POINTS TO BE NOTED:

- **$F = M_{earth}a_{earth} = m_{object}a_{object}$**
- **The value of g has been found on the surface of the earth but is also valid for objects at heights of few meters above the earth surface.**
- **It is independent of the mass of the object.**
- **It depends on the mass and radius of the earth.**

- **The earth also experiences an equal amount of force of attraction towards the object of mass**
  = mg
- **Hence the earth also accelerates towards the object with an acceleration given by**
  $a_{earth} = \frac{mg}{M_{earth}}$
- **The acceleration of the earth is determined by the ratio** $\frac{m}{M_{earth}}$

  This acceleration is too less since (m/M) is very small.

THINK ABOUT THESE

EXAMPLE:

A 100gram apple is falling towards the earth from a height of 10m from the surface of the earth. Find the acceleration with which the earth moves towards the apple. (Mass of the earth = 6 x 10^{24}kg)

SOLUTION:

The distance between the centre of the earth and the apple is the sum of the radius of the earth and the height of 10m which is approximately equal to the radius of the earth = 6.37 x 10^6m. Hence the acceleration due to gravity g for the apple is almost equal to 9.8 m/s^2.

Mass of the earth M= 6 x 10^{24}kg, mass of apple = 100g = 0.1 kg

$$Ma = mg$$

a = $\frac{mg}{M}$ = (0.1 x 9.8) / 6 x 10^{24} = **1.63 x 10^{-25}m/s^2.**

CONCLUSION:

The acceleration of the earth towards the apple is negligibly small. This is the reason why we always see objects falling towards the earth and not the earth rising to meet them. The earth seems to be stationary.

EXAMPLE:

If the point object falling towards the earth from a height of 10m from the surface of the earth is of mass 1 trillion kg (10^{12} kg), what would be the acceleration of the earth towards it?

SOLUTION:

In this case also acceleration of the heavy point mass is taken to be 9.8 m/s^2 as it is only 10m from the surface of the earthand the acceleration of the earth is calculated as:

a = $\frac{mg}{M}$ = $(10^{12} \times 9.8) / 6 \times 10^{24}$ = $\mathbf{1.63 \times 10^{-12}\ m/s^2}$.

Here again the acceleration of the earth is negligible and hence seems to be stationary

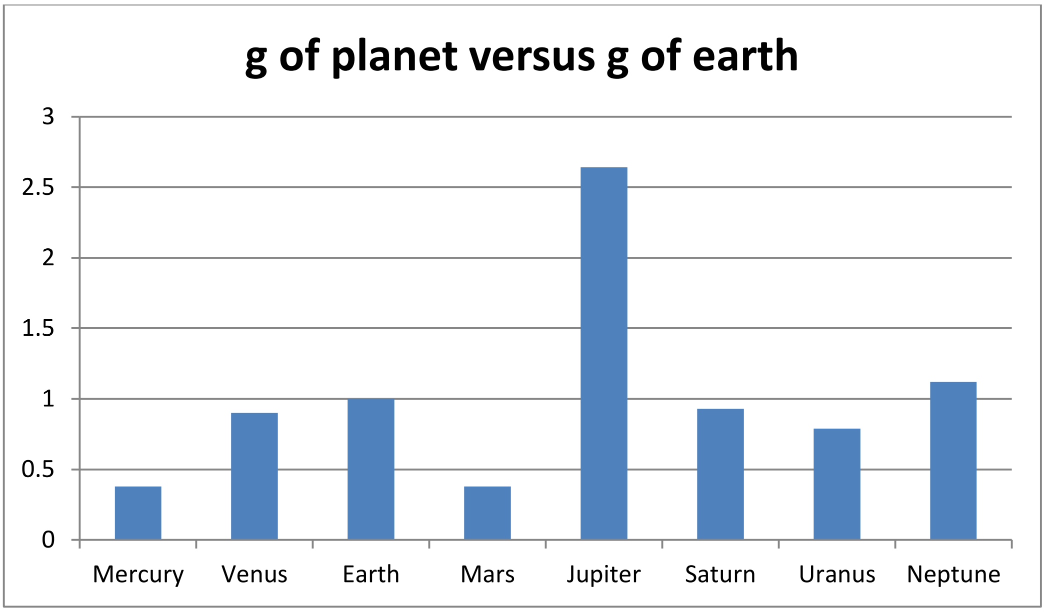
CONCLUSION:

Unless the mass of the object falling towards the earth has a mass comparable to the earth, acceleration of the earth will be too less to be noticeable.

6. ACCELERATION DUE TO GRAVITY (g ‘) ON THE SURFACE OF OTHER PLANETS

The acceleration due to gravity on the surface of the planets of the solar system can be calculated if we know the mass and the radius of the planets.

Planets	Mass (in Earth mass)	Radius (r) (in Earth radius)	r^2	Ratio g’ of planet versus g of earth
Mercury	0.055	0.38	0.14	0.38
Venus	0.815	0.95	0.902	0.90
Earth	1.000	1.00	1	1.00
Mars	0.107	0.53	0.28	0.38
Jupiter	318	10.8	117	2.64
Saturn	95	9.0	81	0.93
Uranus	14.5	3.93	15.4	0.79
Neptune	17.2	3.87	14.9	1.12



From the above table and the graph of g of various planets of the solar system we see that:

- **Value of g on the surface of Jupiter is maximum, so an object will be heaviest on Jupiter**
- **Value of g on Mercury and Mars is same and is the least among the planets of the solar system, so an object will be lightest on these two planets.**
- **Value of g on Mercury and Mars is the same although their masses and radii are very different. Mass of Mars is almost double the mass of mercury and its radius is almost one and half times the radius of Mercury.**
- **Weight of an object would be greater than its weight on earth only on the surface of Neptune and Jupiter. On all the other planets the weight of the object would be smaller than its weight on earth.**

THINK ABOUT THIS

EXAMPLE:

What will the weight of a 100g apple on Earth, Mars and Jupiter? Refer to the table above for the required data.

SOLUTION:

Mass of the apple = 100g = 0.1kg

Weight of an object W = mg(of the planet)

Weight of apple on Earth = 0.1 × 9.8 = 0.98N

Weight of apple on Mars = 0.1 × 0.38 × 9.8 = 0.372N

Weight of apple on Jupiter = 0.1 × 2.64 × 9.8 = 2.6N

7. VARIATION OF ACCELERATION DUE TO GRAVITY (g) WITH HEIGHT

The radius of the earth is 6400000 m which is quite large in comparison to the height to which objects are generally raised from the surface of the earth. Even if an object is placed on top of the Mount Everest which is the highest peak on the earth, it is still only 8848 m high which is also quite small in comparison to the radius of the earth.

So let us consider the variation in acceleration due to gravity g at heights which are quite small in comparison to the radius of the earth.

The value of g at a height h above the earth surface is given by:

$$g' = \frac{GM}{(R+h)^2}$$

$$g' = \frac{GM}{[R\left(1+\frac{h}{R}\right)]^2}$$

$$= \frac{GM}{R^2}\left(1+\frac{h}{R}\right)^{-2}$$

$$= g\left(1+\frac{h}{R}\right)^{-2}$$

If $h \ll R$, using binomial expansion we get the value of g'

$$\therefore g' = g\left(1-\frac{2h}{R}\right)$$

Fractional change in g = $(g-g')/g = \frac{2h}{R}$

Video of Variation of g with height: https://youtu.be/xC8kW7SCEo0

We have found the value of g on the surface(or very near to the surface) of the earth and other planets till now. In these cases the distance between the centre of the earth (or any other planet) and the object is taken to be equal to the radius of the planet.

$$g = \frac{GM}{r^2} \qquad \text{where } r = R \text{ (Radius of the planet)}$$

But if the object is at a substantial height h from the surface of the earth the distance between the centre of the earth and the object can no longer be taken equal to the radius of the earth. Now in the above equation r = R+ h

$$\therefore \qquad g = \frac{GM}{r^2} = \frac{GM}{(R+h)^2}$$

This value of g will be less than the value of g on the surface of the earth.

THINK ABOUT THESE!!

EXAMPLE:

What will be the new value of g experienced by an object which is taken from the surface of the earth to heights equal to?

i) **R**

ii) **2R**

iii) **3R**

iv) **From the above result find the new value of g at a height of nR from the surface of the earth. Here n= natural number (0,1,2,3,4,5 ____)**

SOLUTION:

$$g' = \frac{GM}{(R+h)^2} \quad g = \frac{GM}{R^2}$$

i. At h = R, $g' = \frac{GM}{(R+R)^2} = g/4$

ii. At h =2R, $g' = \frac{GM}{(R+2R)^2} = g/9$

iii. At h =3R, $g' = \frac{GM}{(R+3R)^2} = g/16$

iv. At h =nR, $g' = \frac{GM}{(R+nR)^2} = g/(n+1)^2$

EXAMPLE:

What would be the value of acceleration due to gravity at the surface of astar of mass M = 2 x 10^{31}kg having a radius of 2.95 x 10^{6} m.

SOLUTION:

$$g = \frac{GM}{R^2} = (6.67 \times 10^{-11} \times 2 \times 10^{31}) / (2.95 \times 10^{6})^2 = \mathbf{1.5 \times 10^{8} m/s^2}$$

EXAMPLE:

If an astronaut whose height is h is 1.7m is floating with his feet down in an orbiting space shuttle at a distance of 6.77 x 10^{6}m from the center of this star, what will be the difference between the acceleration due to gravity of the star at her feet and her head?

SOLUTION:

The height of the astronaut is very less in comparison to the orbital radius, so the small change in g can be found by differentiating the equation:

$$g = \frac{GM}{r^2}$$

$$dg = -2\frac{GM}{r^3}dr$$

Substituting r = 6.77 x 10^{6}m, G = 6.67 x 10^{-11}kgm^{-1}s^{-1}, M = 2 x 10^{31} kg and dr = 1.7m

dg= - 10.7m/s^2

So there is a substantial difference in the value of g at the astronaut's feet and head which will be very painful and will cause his body to stretch

EXAMPLE:

What will be the fractional change and the percentage change in the value of g at a height of 340m from the surface of the earth? Take the radius of the earth to be 6400 km.

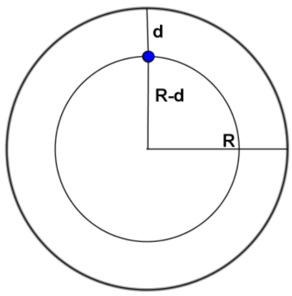
SOLUTION:

Fractional change in g = $\frac{2h}{R}$

$$= \frac{640}{6400000} = \frac{1}{10000}$$

Percentage change = $\frac{1}{10000}$ x 100 = **0.01%**

8. VARIATION OF ACCELERATION DUE TO GRAVITY WITH DEPTH



Newton's law of gravitation talks about the gravitational force between two point masses. But if one of the mass is an extended mass, the gravitational force on the other point mass can be found by vectorially adding the forces experienced by it due to the various point masses of which the extended mass is made of. This is in accordance with the superposition principle.

For one special case when the extended mass is a uniform shell of uniform mass distribution, the force on a point mass inside the shell is given by the **Newton's shell theorem**. According to this theorem:

A uniform shell of matter exerts no net gravitational force on a point mass located inside it.

This is because the forces on the point mass due to various points of the shell add up vectorially to make the net force on it zero.

If we assume the earth to be a perfect sphere of uniform density, then we can use the shell theorem to find the gravitational force and hence the acceleration due to gravity on an object inside the earth.

We can consider the earth to be made up of uniform concentric shells.

If an object is placed inside the earth at a depth d from the earth surface, it experiences a gravitational force only due to the portion of the spherical earth of radius (R – d) below it. It does not experience any force due to the spherical shell of the earth of thickness d above it.

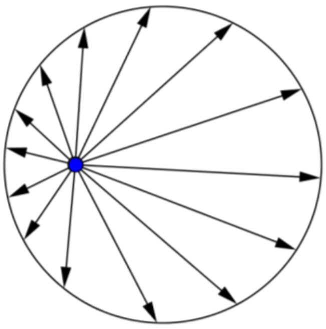
The ratio of the effective mass of the earth which exerts a force on the object to the total mass of the earth is given by:

$$\frac{M_{R-d}}{M_R} = \frac{(R-d)^3}{R^3}$$

$$M_{R-d} = \frac{(R-d)^3 M_R}{R^3}$$

The acceleration due to gravity at the depth d is given by:

$$g' = \frac{G M_{R-d}}{(R-d)^2}$$



$$g' = \frac{G}{(R-d)^2} \frac{(R-d)^3 M_R}{R^3}$$

$$= \frac{G M_R}{R^2} \frac{(R-d)}{R}$$

$$g' = g \frac{(R-d)}{R}$$

Here (R−d) is the distance from the centre of the earth. Let R−d = x, then we have:

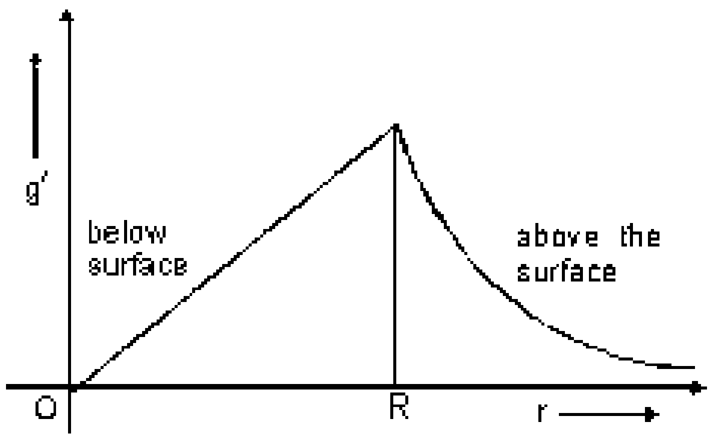
$$g' = g\frac{x}{R} = kx$$

where k is a constant. This shows that the acceleration due to gravity varies linearly with distance from the centre of the earthup to the surface of the earth

Value of k = $\frac{g}{R} = \frac{GM}{R^3} = G(\frac{M}{\frac{4}{3}\pi R^3})(\frac{4}{3}\pi) = G\rho(\frac{4}{3}\pi)$

Plot of the variation of g with distance from the centre of the earth from the following data:

Distance r centre of the earth in terms of radius R of earth	Value of $g^/$
0	0
R/8	1/8g
R/6	1/6g
R/4	1/4g
R/2	1/2g
R	G
2R	g/4
3R	g/9
4R	g/16
5R	g/25



Video of Graphical variation of depth and altitude:

https://youtu.be/u25VerMBTOU

EXAMPLE:

What is thefractional change in the value of g at a depth d?

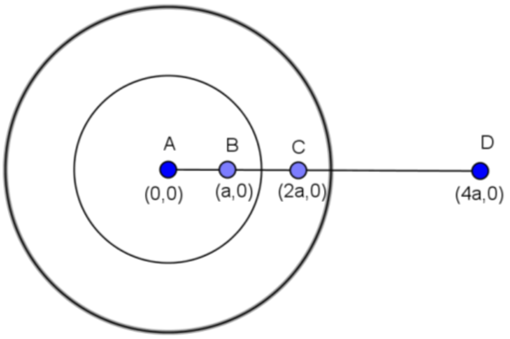
SOLUTION:

$g^/ = g(1 - \frac{d}{R})$

$(g - g')/g = \frac{d}{R}$

EXAMPLE:

Two concentric shells of identical mass M are as shown in the adjacent figure. What is the gravitational force experienced by point objects each of mass m placed at the points A, B, C and D. Co-ordinates of the particles is shown.



SOLUTION:

Force on objects at A and B is zero as theyare inside both the shells.

Force on object C is due to the inner shell of mass M which behaves as if its mass is concentrated at the centre

$$\text{Hence, F(at C)} = \frac{GMm}{4a^2}$$

Force on object D is due to both the shells which behave as their entire mass 2M is concentrated at the centre.

$$\text{Hence, F(at D)} = \frac{\mathbf{GMm}}{\mathbf{16a^2}}$$

9. OTHER FACTORS AFFECTING THE VALUE OF g

1. **Earth does not have a uniform density:**
   Earth has a total mass of 5.98 x 10^{24}kg and a radius of 6370 km. Earth is divided into three zones according to its density.

a. The earth crust::

The outer most layer of the earth has the least density and it has a thickness of 25km from the surface of the earth. The mass of the earth's crust is 3.94 x 10^{22}kg.

b. The mantle:

The layer inside the crust has the highest density and it has a thickness of 2855 km. The mass of this layer is 4.01 x 10^{24}kg.

c. The inner core:

The inner most layer of the earth is a sphere which has a radius of 3490 km. This mass of this layer is 1.93 x 10^{24}kg

The value of g at the interface of these layers can be calculated using the shell theorem and it is found that its value actually increases with depth for some time before it starts decreasing.

2. **Earth is not a perfect sphere:**

Earth is not a perfect sphere but an ellipsoid, flattened at the poles and bulging at the equator. Its equatorial radius is greater than its polar radius by 21km. Hence a point at the poles is closer to the centre of the earth than a point on the equator.
So the
value of g is higher at the poles as compared to that at the equator.

3. **Rotation of the earth:**

The earth rotates about its own axis. So an object on the surface of the earth is also rotating with the angular velocity ω of the earth in circles of different radii depending on the latitude of the earth. The object at the equator is making largest circle which decreases as we move towards the pole. At the poles the object is almost stationary.

The variation in the value of g due to rotation is given by:

$$\mathbf{g' = g - \omega^2 R}$$

where R is radius of circle at various latitudes.
At the equator this difference in g' and g is 0.034m/s^2.

10. SUMMARY

- **Acceleration due to gravity (g)**: It is the acceleration experienced by an object due to the gravitational force of attraction of the earth (or any other planet). On the surface of the earth (or any planet), it depends on the mass of the planet and the radius of the planet. It is independent of the mass of the object

- **Variation of acceleration due to gravity(g):**

With height (altitude): The value of g decreases with height from the surface of the earth. It varies inversely as the square of the distance outside the surface of the earth

With depth: If the earth is considered to be a uniform sphere, the value of g decreases linearly with depth from the surface. Its value is zero at the centre of the earth.

This result follows from the shell theorem that the net gravitational force of attraction on a point object is zero inside a spherical shell of uniform density.

With latitude: Due to the rotation of the earth about its axis, value of g decreases from the poles to the equator.